Sovereign Default, TFP, Fiscal and Financial Frictions*

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Abstract

We extend the prototypical model of debt repudiation to simulate the observed evolution of key economic and financial aggregates. Our analysis rests on a novel numerical algorithm for the computation of sustainable debt paths. We introduce physical capital accumulation and endogenous labor supply, optimal fiscal policy, and partial default. We model the cost of default as a persistent loss in TFP that may even arise prior to default. We also address some fiscal and financial frictions neglected in the literature. Most models of sovereign debt are not suitable to analyze the long recessionary periods around the time of default, and may generate rather low debt-to-GDP ratios and high credit spreads.

JEL Classification: F3, F4.

Keywords: Foreign Debt, Sovereign Default, Haircut, Credit Spread.

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1 Introduction

In this paper we present a quantitative framework for sovereign default. We are motivated by the lack of available models that can replicate the behavior of economic aggregates around these default episodes. As discussed below, most well known extensions of the prototypical model of debt repudiation [Eaton and Gersovitz (1981)] tend to generate rather low debt-to-GDP ratios at the time of default, and rather high credit spreads. Moreover, as most of these models abstract from production activities, they are not suitable to analyze these regular long recessionary periods around the time of default characterized by persistent drops in productivity, the physical capital stock, and employment.

On these grounds, we extend the basic model of Eaton and Gersovitz (1981) along several dimensions. We consider a production economy with physical capital and labor to assess these long declines in economic activity associated with default episodes. We model the cost of default as a persistent loss in total factor productivity (TFP). This reduced-form representation of the cost of default provides a useful benchmark for evaluating related micro-based formulations, economic frictions, and institutional considerations. Also, the sovereign may tax income to provide for public consumption and pay for outstanding debt obligations. Hence, our model becomes instrumental to analyze economic policies for avoiding default and their influence on the macroeconomy. Most notably, the sovereign may impose a haircut on its foreign creditors. We consider various scenarios for partial debt repudiation and study their impact on real and financial variables.

These extensions significantly complicate our quantitative analysis as compared to related models in the literature. Following Kydland and Prescott (1980) and Feng et al. (2014), we develop a numerical algorithm that can accurately approximate all the equilibrium solutions over the set of sustainable debt paths. Physical capital adds an additional endogenous state variable. Fiscal policy imposes extra non-linear constraints on our recursive contract problem for the government as we deal with the problem of time-inconsistency over succeeding ruling entities. External debt haircuts make it harder to identify the set of sustainable plans; this set is most easily approximated under full default or internal debt. We shall impose some simplifying assumptions as we combine all these ingredients into an abstract computable framework.

We first present a baseline economy and evaluate the dynamics of output, investment, employment, the debt-to-GDP ratio, and the sovereign bond spread to changes in parameter values. As in Geanakoplos and Zame (2014), sovereign default acts as insurance against bad states, and it may be welfare improving. But it could have a great impact on trading and
pricing. Hence, complete debt repudiation seems a rather awkward arrangement. Indeed, in this case the debt-to-GDP ratio plummets and the sovereign bond spread increases considerably. In our setting the notion of trying to match observed risk premia to the data under complete default would seem questionable. In our database the mean value of the haircut is about 38 percent. On the other hand, a smaller TFP loss as a consequence of default may result in a lower debt-to-GDP ratio, and a decrease in the credit spread. In some cases, the change in the credit spread may be less perceptible because under a smaller TFP loss the sovereign will default more often and is less motivated to run a fiscal surplus to avoid the default cost. In contrast, the persistence of the TFP loss affects the sovereign bond spread substantially. Long declines in TFP are also needed to account for these observed swings in real and financial variables. In fact, a better fit for the model to the data is obtained when the TFP loss starts to happen before default. As in some financial models discussed below, this premonitory fall in macroeconomic activity could reflect fears of the worsening of credit conditions and investment. Traditional models of sovereign debt simply assume that output costs occur after the event of default.

Our optimization framework calls for prudent fiscal policies at the time of default. While the sovereign is motivated to run budget surpluses to prevent default, in our dataset we observe deficits with flat tax revenues and flat general government consumption. Following Conesa and Kehoe (2017), this empirical anomaly can be thought of “betting for redemption”. In our model, fiscal deficits at the time of default would be observed in situations of complete default, and for rather recessionary conditions with low persistent and minor default costs. These expansionary policies result in lower debt-to-GDP ratios and higher credit spreads.

Our basic model is also unable to account for another basic fact: internal debt carries a higher risk premium before default and external debt carries a higher risk premium after default. This latter empirical anomaly may originate from a financial friction as a reflection of the “investment home bias” puzzle, which could be triggered by a credibility loss on the part of foreign investors. In reality, the optimal debt mix of the sovereign is tilted towards internal debt after the event of default.

All of these issues highlight some of the controversies of the modeling of default in economic theory. Dynamic stochastic general equilibrium models operate on simple working assumptions, and usually abstract from reputation and institutional considerations. These general equilibrium models, however, can be sampled over a wide range of parameter values, and hence their results can be extrapolated to many situations of interest.
Much of the economics literature on sovereign debt starts with Eaton and Gersovitz (1981). This paper considers an endowment economy in which default is punished through permanent exclusion from international capital markets. In this model the sovereign may be tempted to default in good states; moreover, a permanent shut-down of international credit would then call for a complete debt write-off. Arellano (2008) and Aguiar and Gopinath (2006) introduce exogenous endowment risk. Defaults are punished by a temporary separation from international markets and a convex output loss. This helps to generate countercyclical sovereign bond spreads. Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012) and Hatchondo, Martinez, and Sosa-Padilla (2016) endogenize the maturity structure of sovereign debt, and examine various channels for long-term debt to influence the level and volatility of the credit spread.

There are mixed views on the importance of imposed restrictions to trade and capital flows as a result of sovereign default [Martinez and Sandleris (2011) and Rose (2005)]. As in many other papers in macroeconomics, we consider that the ensuing costs of separation from external credit are of second-order importance compared to the disruptive effects to the domestic economy (e.g., insolvency, more restricted access to domestic credit, banking crises, and currency depreciation). A more recent literature has introduced endogenous default costs, which may be magnified by several propagation mechanisms from primary shocks to the domestic economy. Again, in these papers physical capital investment and partial debt repudiation are not considered because of analytical tractability. Mendoza and Yue (2012) postulate a working capital constraint for importing intermediate goods. Sovereign default will limit access to international credit, and hence diminishes domestic firms’ ability to import superior international intermediate goods. In Na et al. (2018), nominal wage rigidities will lead to involuntary unemployment. Then, devaluation of the currency is necessary to stimulate production of local goods. In Bocola (2016), government default causes a deterioration of the banking sector’s balance sheet, which in turn affects the credit condition of the private sector and the production capacity of the domestic economy. Perez (2019) and Arellano, Bai, and Bocola (2017) present other models of domestic banking and default. Gordon and Guerron-Quintana (2018), and Park (2017) study production economies with full debt repudiation. Partial default is only introduced in Arellano, Mateos-Planas, and Rios-Rull (2019) in an exchange economy, and in Pei (2019) in a production economy with no external financing. Their analytical frameworks differ substantially from ours.

The paper is organized as follows. In Section 2 we document some well established
empirical regularities for real and financial variables upon the occurrence of sovereign default. Section 3 introduces the basic model, and Section 4 presents the numerical algorithm for the computation of equilibrium solutions. Section 5 considers our baseline economy, and explores some quantitative implications. In Section 6 we perform some comparative statics exercises. Section 7 discusses some further extensions of the baseline economy. Section 8 concludes.

2 Some Empirical Regularities

The Appendix collects our data sources. We gather data from 16 default events in the 1990-2010 period. We choose this time period because the default episodes of the 1990s represent a substantial departure from historical experience [cf. Chuhan and Sturzenegger (2005)]. The development of international financial markets has changed the landscape of country risk exposure. By and large, these sovereign defaults have not heavily impacted the international financial system, and most debt resolutions have been completed fairly quickly within a year.

Sovereign default is defined as either the failure to meet a principal or interest payment on the due date of the original terms of a debt contract or as an exchange offer of new debt with less favorable terms than the original issue [Beers and Chambers (2006), Cruces and Trebesch (2013)]. We get TFP data, GDP, consumption, current account balance, fiscal balance, investment, government spending, taxation, and the real interest rate from the World Bank Open Data. The debt structure data is provided by Panizza (2008), whose study covers developing countries.\footnote{In Panizza (2008), the external debt is defined as the debt owned by non-residents. This is consistent with the definition adopted by The External Debt Statistics: Guide for Compilers and Users, published by BIS, Eurostat, IMP, OECD, and the World Bank.} We supplement this data with Reinhart and Rogoff (2011). The aggregate sovereign bond spread is obtained from the J. P. Morgan Emerging Markets Bond Index (EMBI+) for Argentina (1994Q1-2002Q2) and Ecuador (1996Q2-1999Q2). For all the other countries in our sample, we obtain the aggregate sovereign bond spread from interest data of the Database of Fiscal Space by Kose et al. (2017).

Figure 1 reports the average behavior of these aggregate variables across all the default episodes in our sample\footnote{Due to space consideration the same graphs for individual countries are not reported; they are, however, available on request.} within a five-year window before and after the time of default (time $t = 0$).
Fact 1: Macroeconomic activity: Default often occurs in bad times. Capital accumulation, employment and TFP mainly decline during the five years before default. Over this time period, the GDP level shrinks gradually up to 5.1 percent on average, and keeps declining after default. The share of total investment in GDP declines before default, and then does not pick up completely; the total private consumption share stays roughly constant. The current account balance remains flat around the time of default.

Fact 2: Fiscal policy: The ratio of tax revenue to GDP decreases from 14.2 percent to 13.5 percent on average at the time of default, and increases to 14.5 percent after default. The ratio of general government consumption to GDP rises slowly from about 14 percent to about 15 percent at the time of default, and then decreases after default. Hence, there is an observed primary fiscal deficit before default, and a primary fiscal surplus after default.

Fact 3: Sovereign debt: The total debt-to-GDP ratio increases to 78 percent at the time of default. Defaults usually come in the form of debt write-downs rather than complete debt cancellations. The average haircut is about 38 percent. The debt-to-GDP ratio is negatively correlated with output growth, and positively correlated with the tax revenue and the current account balance. The total debt mix of the sovereign tilts toward internal debt after default.

Fact 4: Sovereign bond spread: The average sovereign bond spread increases to about 3,000 basis points for domestic debt and 4,000 basis points for external debt. Only before default does domestic debt bear a higher interest rate. The sovereign bond spread is negatively correlated with GDP growth and with the ratio of domestic-to-external debt, and positively correlated with total debt and inflation.

Fact 5: Sovereign default across countries: There is a great variation of total debt-to-GDP ratios and sovereign bond spreads across countries. Some less developed economies have been exposed to repeated defaults.

Fact 1 redefines our modeling approach: there is a substantial TFP loss that happens before default. We need to pick up this early TFP loss to mimic the evolution of GDP, capital accumulation, and employment. Regarding Fact 2, our analysis points to the existence of a fiscal friction: optimal fiscal policies should be characterized by pronounced budget surpluses.
Figure 1: Economic dynamics around default
to avoid default. “Betting for redemption” could just be justified at times of complete default, severe output contractions, and very temporary and rather low costs of default.

We do not attempt to provide a theory of the haircut, which is about 38 percent; see Fact 3. In our model the size of the haircut has a great impact on the debt-to-GDP ratio and the sovereign bond spread. We have also mentioned the possible existence of a financial friction: external debt becomes more expensive after default. Hence, Fact 4 hints at some kind of “investment home bias” after default.

Regarding Fact 5, we are not searching for a baseline economy that fits all default episodes, since there could be a great variability of the costs of default across countries. While Japan can bear a sovereign debt-to-GDP ratio of about 260 percent under a very low credit spread, some less developed economies may be exposed to repeated defaults. We will sample over the parameter space to explore the determinants of the debt-to-GDP ratio and the sovereign bond spread.

Table 1: Determinants of the total debt-to-GDP ratio

<table>
<thead>
<tr>
<th>Regression</th>
<th>Output growth</th>
<th>Domestic/external debt</th>
<th>Inflation</th>
<th>Tax revenue/GDP</th>
<th>Current account balance/GDP</th>
<th>Exports/GDP</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output growth</td>
<td>(-0.83^*) (0.15)</td>
<td>(-1.21^*) (0.21)</td>
<td>(-1.29^*) (0.28)</td>
<td>(-1.82^*) (0.29)</td>
<td>(-1.84^*) (0.30)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic/external debt</td>
<td>(0.25^*) (0.07)</td>
<td>(-0.12) (0.25)</td>
<td>(-0.15) (0.19)</td>
<td>(-0.13) (0.20)</td>
<td>(0.64^*) (0.22)</td>
<td>(0.58^*) (0.23)</td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>(0.66) (0.35)</td>
<td>(-0.40) (0.32)</td>
<td>(-0.56) (0.33)</td>
<td>(0.74^*) (0.24)</td>
<td>(-0.014) (0.075)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax revenue/GDP</td>
<td>(0.64^*) (0.22)</td>
<td>(0.58^*) (0.23)</td>
<td>(0.74^*) (0.24)</td>
<td></td>
<td>(-0.014) (0.075)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current account balance/GDP</td>
<td>(0.64^*) (0.22)</td>
<td>(0.58^*) (0.23)</td>
<td>(0.74^*) (0.24)</td>
<td></td>
<td>(-0.014) (0.075)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exports/GDP</td>
<td>(-0.014) (0.075)</td>
<td></td>
<td></td>
<td>(-0.014) (0.075)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2312</td>
<td>1693</td>
<td>919</td>
<td>651</td>
<td>622</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.17</td>
<td>0.20</td>
<td>0.21</td>
<td>0.26</td>
<td>0.27</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We would like to discuss some other issues which bear upon the disruptive costs of default to the domestic economy. Tables 1 and 2 aim to explore the determinants of the debt-to-GDP

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3 The most recent Argentinean peso and debt crises evidenced rather small costs of default and productivity losses because of the dollarization of the economy and their relatively low dependency on the domestic banking system. Likewise, in the past European debt crisis, some peripheral countries (e.g., Portugal, Ireland, Greece, and Spain) saw dramatic increases in their sovereign debt spreads. While their debt-to-GDP ratios were quite commensurate to the core countries (Germany and France), investors may have perceived higher probabilities of separation from the Euro area for these satellite economies.
ratio and the sovereign bond spread for external debt over a broad sample of 115 countries, including all major OECD and emerging market economies. We should point out that both the debt-to-GDP ratio and the sovereign bond spread are strongly correlated with GDP growth, tax revenue, and inflation; however, their correlation with trade variables is not robust. This evidence then plays in favor of theories of the cost of default based on domestic productivity. High inflation rates and low taxation may originate from weak governing institutions, which in turn may translate into less developed banking and credit systems and the dollarization of these economies. Asonuma (2016) (Table 1) reports a strong positive correlation between the credit default swap spread and inflation, but a weak correlation with capital account openness. Along similar lines, Reinhart, Rogoff, and Savastano (2003) write: *The lower costs of financial disruption that these countries face may induce them to default at lower thresholds, further weakening their financial systems and perpetuating the cycle.*

One might make the same comment about tax systems.

Table 2: Determinants of the sovereign bond spread for external debt

<table>
<thead>
<tr>
<th>Regression</th>
<th>Output growth</th>
<th>Total debt/GDP</th>
<th>Domestic/external debt</th>
<th>Service of total debt/GDP</th>
<th>Inflation</th>
<th>Tax revenue/GDP</th>
<th>Current account balance/GDP</th>
<th>Exports/GDP</th>
<th>Observations</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-47.59^*$</td>
<td>2.44</td>
<td>$-2.95$</td>
<td>1.69</td>
<td>20.79*</td>
<td>$-19.72$</td>
<td>$-13.57$</td>
<td>-2.64</td>
<td>356</td>
<td>0.194</td>
</tr>
<tr>
<td></td>
<td>(8.14)</td>
<td>(1.49)</td>
<td>(1.83)</td>
<td>(9.53)</td>
<td>(4.34)</td>
<td>(10.63)</td>
<td>(9.92)</td>
<td>(2.09)</td>
<td>253</td>
<td>0.237</td>
</tr>
<tr>
<td></td>
<td>$-52.16^*$</td>
<td>9.81*</td>
<td>$-61.41^*$</td>
<td>3.20</td>
<td>28.90*</td>
<td>$-19.72$</td>
<td>$-13.57$</td>
<td>-2.64</td>
<td>180</td>
<td>0.353</td>
</tr>
<tr>
<td></td>
<td>(10.60)</td>
<td>(2.48)</td>
<td>(22.77)</td>
<td>(19.14)</td>
<td>(3.81)</td>
<td>(10.63)</td>
<td>(9.92)</td>
<td>(2.09)</td>
<td>127</td>
<td>0.495</td>
</tr>
<tr>
<td></td>
<td>$-43.83^*$</td>
<td>11.48*</td>
<td>$-97.74^*$</td>
<td>$-32.05$</td>
<td>$30.70^*$</td>
<td>$-19.72$</td>
<td>$-13.57$</td>
<td>-2.64</td>
<td>127</td>
<td>0.505</td>
</tr>
<tr>
<td></td>
<td>(16.02)</td>
<td>(3.15)</td>
<td>(38.10)</td>
<td>(19.14)</td>
<td>(4.07)</td>
<td>(10.63)</td>
<td>(9.92)</td>
<td>(2.09)</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$-11.84$</td>
<td>9.00*</td>
<td>$-55.12^*$</td>
<td>$-10.93$</td>
<td>-19.72</td>
<td>$-19.72$</td>
<td>$-13.57$</td>
<td>-2.64</td>
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<td></td>
<td>(14.72)</td>
<td>(2.43)</td>
<td>(27.17)</td>
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<td>(10.63)</td>
<td>(9.92)</td>
<td>(2.09)</td>
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<tr>
<td></td>
<td>$-10.34$</td>
<td>7.81*</td>
<td>$-63.35^*$</td>
<td>$-7.01$</td>
<td>$-20.10$</td>
<td>$-19.72$</td>
<td>$-13.57$</td>
<td>-2.64</td>
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<td></td>
<td>(14.74)</td>
<td>(2.55)</td>
<td>(27.65)</td>
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<td>(10.63)</td>
<td>(9.92)</td>
<td>(2.09)</td>
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</tbody>
</table>
3 The Model

Our small-open economy is populated by a continuum of identical households. The representative household starts the economy with \( k_0 \) units of physical capital and can supply up to one unit of labor, \( l_t \), at every date \( t = 0, 1, \cdots \). The sovereign (also called the government) maximizes the country’s welfare, and may impose a haircut, \( 0 < \kappa < 1 \), on its creditors. Upon default, the economy suffers a temporary productivity loss and a one-time utility loss. This latter non-pecuniary penalty will subsequently be replaced by an added wedge over the sovereign bond spread.

The production sector rents capital, \( K_t \), and labor, \( L_t \), from households under a constant-returns-to-scale production technology, \( Y_t = A_t F(K_t, L_t) \), where \( A_t \) is a stochastic variable representing total factor productivity (TFP). The production sector maximizes profits, \( \Pi_{Y,t} = A_t F(K_t, L_t) - R_t K_t - w_t L_t \) at every \( t \); hence, factor prices equal their respective marginal productivities, \( R_t = A_t F'_K(K_t, L_t) \), and \( w_t = A_t F'_L(K_t, L_t) \), at an interior solution. Physical capital is subject to a constant depreciation factor, \( 1 > \delta > 0 \). Output, \( Y_t \), can be devoted to private consumption, \( c_t \), public consumption, \( G_t \), investment, \( i_t \), and exports to pay for the external debt, \( NX_t \). That is, \( Y_t = c_t + G_t + i_t + NX_t \).

The sovereign taxes income at a flat rate, \( \tau_t \), to finance public consumption, \( G_t \), and can borrow a one-period bond from the international lending sector. The sovereign may decide to honor \( \Delta = 0 \) or not to honor \( \Delta = 1 \) the existing debt, \( B_t \). New debt issuance, \( B_{t+1} \), prices at discount, \( q_t \), reflecting the perceived country’s risk. Upon default, the sovereign would only pay for a fraction \( 1 - \kappa \) of its debt balance, whereas productivity \( A_t \) declines by a factor \( \zeta(A_t) \). This productivity loss is lifted with probability, \( \pi_A \), at every future date, \( t \). The government also suffers a one-time utility loss, \( v_t \), which later will be interpreted as an added borrowing cost upon default. The international lending market is perfectly competitive at a constant interest rate, \( \tilde{r} > 0 \). Foreign creditors are risk neutral and maximize expected profits, \( \Pi_{f,t} = \frac{E[(1-\Delta_{t+1})B_{t+1}]}{1+\tilde{r}} - q_t \cdot B_{t+1} \); hence, in equilibrium the following no-arbitrage condition must be satisfied: \( q_t = \frac{E[(1-\Delta_{t+1})]}{1+\tilde{r}} \). Observe that \( q_t \) depends on \( B_{t+1} \), since the sovereign may default in certain states.

3.1 The representative household

For a given sequence of taxes and public consumption \( (\tau_t, G_t)_{t=0}^{\infty} \) and factor prices \( (R_t, w_t)_{t=0}^{\infty} \), the representative household chooses an optimal plan for private consumption, hours worked,
and capital accumulation \((c_t, l_t, k_{t+1})_{t=0}^{\infty}\) to maximize the inter-temporal utility function:

\[
U(c, G, l) = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t [u(c_t, G_t) - h(l_t)] \right\}
\] (1)

subject to the income and resource constraints:

\[
[(1 - \tau_t) \cdot (R_t - \delta) + \delta] \cdot k_t + (1 - \tau_t) \cdot w_t \cdot l_t \geq i_t + c_t, \\
(1 - \delta) \cdot k_t + i_t \geq k_{t+1}.
\] (2) (3)

One-period utility functions \(u\) and \(h\) are assumed to satisfy standard regularity conditions for the utility of both types of consumption, \(c_t\) and \(G_t\), and the disutility of labor supplied, \(l_t\). As is well known, at the optimal path the following transversality condition must hold:

\[
\lim_{t \to \infty} \mathbb{E} \left( \frac{k_{t+1}}{\prod_{s=1}^{t} (1 + r_s)} \right) = 0, \text{ where } r_t = R_t - \delta.
\]

Besides this transversality condition, we also assume that the optimal behavior of the household can be characterized by the first-order conditions:

\[
u_{c_t} = \beta \mathbb{E} [1 + (1 - \tau_{t+1}) \cdot (R_{t+1} - \delta)] \cdot u_{c_{t+1}}
\] (4)

\[
(1 - \tau_t) \cdot w_t \cdot u_{c_t} = h_{l_t},
\] (5)

where \(u_{c_t}, u_{c_{t+1}}\) are the partial derivatives of \(u\) with respect to private consumption, \(c\), at times \(t\) and \(t + 1\), and \(h_{l_t}\) is the marginal disutility of labor.

3.2 The sovereign

The government’s problem is to choose an optimal plan for public consumption, the tax rate, the decision to default, and the quantity of external bond holdings \((G_t, \tau_t, \Delta_t, B_{t+1})_{t=0}^{\infty}\) to maximize the country’s welfare net of a non-pecuniary penalty \(v_t\):

\[
\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t [u(c_t, G_t) - h(l_t) - \Delta_t \cdot v_t] \right\}
\] (6)

subject to budget balance

\[
G_t + (1 - \Delta_t \kappa) \cdot B_t \leq q_t \cdot B_{t+1} + T_t,
\] (7)

for tax revenues, \(T_t = \tau_t \cdot (R_t - \delta) \cdot k_t + \tau_t \cdot w_t \cdot l_t\), and for given feasible paths \((c_t, l_t, k_{t+1})_{t=0}^{\infty}\).
for the representative household.

As is well known [cf. Hernandez and Santos (1996)], the existence of an optimal solution for the government implies that the sum of the discounted stream of tax revenues must be well defined: \( E \left( \sum_{t=0}^{\infty} \frac{T_t}{(1+r)^t} \right) < \infty. \) Hence, every sustainable debt path must obey the transversality condition: \( \lim_{t \to \infty} E \left( \sum_{t+1}^{\infty} \frac{B_{t+1}}{(1+r)^{t+1}} \right) = 0. \)

### 3.3 Competitive equilibrium

**Definition 3.1. (Competitive Equilibrium)** For given initial values \((A_0, k_0, K_0, B_0)\), a competitive equilibrium with the possibility of default for our small-open economy is a sequence of prices \((R_t, w_t, q_t)_{t=0}^{\infty}\), quantities \((c_t, k_{t+1}, l_t, K_t, L_t)_{t=0}^{\infty}\), and government choices \((G_t, \tau_t, \Delta_t, B_{t+1})_{t=0}^{\infty}\) satisfying the following conditions:

1. **The Representative Household:** \((c_t, k_{t+1}, l_t)_{t=0}^{\infty}\) maximizes the objective (1) subject to the income and resource constraints (2)-(3), for the given prices and government policies;

2. **The Sovereign:** \((G_t, \tau_t, \Delta_t, B_{t+1})_{t=0}^{\infty}\) maximizes the objective (6) subject to budget balance (7), for the given prices and household’s optimal choices, with \( q_t = \frac{E(1-\Delta_{t+1})}{1+r} \);

3. **The Production Sector:** \((K_t, L_t)\) maximizes profits \(\Pi_{Y,t}\) at every date \(t\);

4. **The Foreign Lending Sector:** \(B_{t+1}\) maximizes profits \(\Pi_{f,t}\) at every date \(t\);

5. **Market Clearing:** \(Y_t = c_t + G_t + i_t + NX_t\), for \(NX_t = (1-\Delta_t) \cdot B_t - q_t \cdot B_{t+1}\), and \(K_t = k_t, L_t = l_t\), at every date \(t\).

In what follows, we assume that factor prices are always equal to their marginal productivities: \(R_t = A_t F_k(K_t, L_t)\), and \(w_t = A_t F_l(K_t, L_t)\).

### 4 Recursive Equilibria: A Numerical Algorithm

The above competitive equilibrium runs into a well-known problem of time-inconsistency: every future government may disavow existing debts. Besides, we have to rely on an equilibrium concept amenable to computation.\(^4\) We focus on a time-invariant recursive equilibrium that depends on pay-off relevant states.

---

\(^4\)See Feng (2015) for a discussion of some equilibrium concepts.
4.1 Markov perfect equilibrium

For notational convenience, we drop the time subscript $t$ whenever possible, and $x_+$ will denote the next future value of every variable $x$.

Recursive representation of equilibrium for the private sector: Every household observes the state of the economy: $\theta = (A, k, K, B)$, and perceives a set of time-invariant policies from the government: $\tilde{\Theta} = (G(\theta), \tau(\theta), \Delta(\theta), B_+(\theta))$, the transition function for state variables $(A_+, K_+) = \Phi(\theta)$, as well as the path of asset and factor prices $(R(\theta), w(\theta), q(\theta))$, depending on the vector of states of the economy $\theta$. Equilibrium of the private sector requires market clearing for the aggregate good and factor markets at all possible states of the economy $\theta$. The maximization problem of the household is represented as follows:

$$V(\theta; \tilde{\Theta}) = \max_{\{c,k,l\}} \left\{ u(c, G) - h(l) + \beta E V(\theta_+; \tilde{\Theta}_+) \right\}$$  \hspace{1cm} (8)

subject to

$$[(1 - \tau) \cdot (R - \delta) + \delta] \cdot k + (1 - \tau) \cdot w \cdot l \geq i + c$$  \hspace{1cm} (9)
$$ (1 - \delta) \cdot k + i \geq k_+$$  \hspace{1cm} (10)
$$ (A_+, K_+) = \Phi(\theta).$$  \hspace{1cm} (11)

Profit maximization implies:

$$R = AF_k(K, L)$$  \hspace{1cm} (12)
$$w = AF_l(K, L).$$  \hspace{1cm} (13)

And market clearing implies:

$$l = L$$  \hspace{1cm} (14)
$$k = K$$  \hspace{1cm} (15)
$$c + i + G + NX = AF(K, L).$$  \hspace{1cm} (16)

Recursive representation for the sovereign: Let $\left( \hat{c}(\theta; \tilde{\Theta}), \hat{k}_+ (\theta; \tilde{\Theta}), \hat{l} (\theta; \tilde{\Theta}); \hat{V}(\theta; \tilde{\Theta}) \right)$ be an equilibrium solution to problem (P-1), for any given set of perceived government
policies $\tilde{\Theta}$, defining a corresponding value function for the sovereign, $\tilde{W}(\theta; \tilde{\Theta})$. Then, the sovereign chooses an optimal policy $\Theta = (G, \tau, \Delta, B_+)$ to maximize social welfare

\[
\max_{\{G, \tau, \Delta, B_+\}} \left\{ u(\hat{c}, G) - h(\hat{l}) - \Delta v + \beta \mathbb{E} \tilde{W}(\theta_+; \tilde{\Theta}_+) \right\}
\]  

subject to feasibility and budget balance:

\[
\hat{c} + \hat{i} + G + NX \leq AF(K, \hat{L})
\]

\[
G + (1 - \Delta \kappa) \cdot B \leq q(\theta, B_+) \cdot B_+ + \tau \cdot (AF(K, \hat{L}) - \delta \cdot K)
\]

\[
q(\theta) = \frac{\mathbb{E} (1 - \Delta \kappa)}{1 + \tilde{r}}.
\]

Observe that the sovereign adjusts fiscal variables rather than resource allocation, and the representative household follows upon these new policies to select optimal choices. Hence, (P-1)-(P-2) conform a sequential game in which the government moves first, and so every perceived policy $\tilde{\Theta}$ generates a competitive equilibrium for the private sector. Also, the current sovereign takes into account the independent actions of future governments. This resolves the time-inconsistency problem because incoming ruling entities only care about the given state of the economy without further consideration to past policies.

**Definition 4.1.** (Markov Perfect Equilibrium (MPE)) A Markov Perfect Equilibrium for the above economy consists of a set of policy functions $\left(c \left( \theta; \tilde{\Theta} \right), k_+ \left( \theta; \tilde{\Theta} \right), l \left( \theta; \tilde{\Theta} \right) \right)$, government policies $\Theta(\theta) = (G(\theta), \tau(\theta), \Delta(\theta), B_+(\theta))$, price functions $(R(\theta), w(\theta), q(\theta))$, perceived policies $\tilde{\Theta}(\theta)$, and value functions, $V(\theta; \tilde{\Theta})$ and $W(\theta; \tilde{\Theta})$, such that

1. $\left(c \left( \theta; \tilde{\Theta} \right), k_+ \left( \theta; \tilde{\Theta} \right), l \left( \theta; \tilde{\Theta} \right) \right)$ solves the representative household’s problem (P-1) for the given prices and perceived policies;

2. $\Theta(\theta)$ solves the government’s problem (P-2) for the given prices and household’s optimal choices;

3. $(r(\theta), w(\theta), q(\theta, B_+))$ satisfy (12), (13) and (20);

4. The aggregate good and factor markets clear at all times, and hence conditions (14), (15) and (16) are satisfied at every $t$;

5. Perceived policies are identical to actual ones: $\tilde{\Theta} = \Theta$. 

14
Proposition 4.2. Let $B \in \mathbf{B}(A, K)$ be the set of MPE debt stocks $B$ for the sovereign for initial conditions $A$ and $K$. Assume that production function $F(K, L)$ is concave, and $\lim_K F_k(K, L) < \delta$ for all $L$. Then, $\mathbf{B}(A, K)$ is a compact set.

Proposition 4.3. For the economy described in Section 3, an MPE is characterized by the following conditions:

(P-3)\[
W(\theta, \Theta) = \max_{\{\Delta, G, \tau, B_+\}} \{u(c, G) - h(l) - \Delta u + \beta \mathbb{E} W(\theta_+; \Theta_+)\}
\]
subject to

\[
c + i + G + (1 - \Delta \kappa) \cdot B \leq AF(k, l) + \frac{\mathbb{E}(1 - \Delta_+ \cdot \kappa)}{1 + \bar{r}} \cdot B_+\tag{22}
\]
\[
(1 - \delta) \cdot k + i \geq k_+\tag{23}
\]
\[
G + (1 - \Delta \kappa) \cdot B \leq \frac{\mathbb{E}(1 - \Delta_+ \cdot \kappa)}{1 + \bar{r}} \cdot B_+ + \tau \cdot (AF(k, l) - \delta \cdot k)\tag{24}
\]
\[
u_c = \beta \cdot \mathbb{E} \left[1 + (1 - \tau_+) \cdot (A_+ F_{k+} - \delta)\right] \cdot u_{c_+}\tag{25}
\]
\[
(1 - \tau) \cdot AF_l \cdot u_c = h_l\tag{26}
\]
\[
B_+ \in \mathbf{B}(A_+, k_+),\tag{27}
\]

where $\Theta_+ \equiv \Theta(A_+, K_+, B_+)$, and $\Theta_+ = (G_+, \tau_+, \Delta_+, B_{++})$.

4.2 The numerical algorithm

In our quantitative analysis, we must impose workable debt limits to speed up the computation of optimal policies, but sufficiently lax not to misrepresent the set of policies consistent with sustainable debt dynamics. Partial default, external debt, endogenous fiscal policy, as well as production and investment make these sustainable debt plans much less tractable. More specifically, with complete default ($\kappa = 1$), regardless of the level of external debt inherited from the previous period, the government can always satisfy the resource constraint (24). Under a predetermined haircut, $0 < \kappa < 1$, however, there may not exist a viable government policy choice that satisfies the resource constraint (24) when $B$ is sufficiently large. Likewise, if the stock of sovereign debt is entirely held by domestic households [cf. Bocola (2016), and Pei (2019)], the government can always dispose of the debt burden by taxing income at a sufficiently high rate. Tight debt bounds are also harder to establish.
with production and investment. In our case, these bounds will depend on fiscal policies to be determined by future governments. With exogenous endowment processes, one may find the natural debt limit by calculating the present value of the total endowment.\footnote{For example, Mendoza and Yue (2012) define the debt limit as $\bar{y}/r$, while Arellano (2008) assumes an exogenous borrowing limit $Z$ for the government.} For an endogenous haircut (as in some formulations with Nash-bargaining; e.g., Yue (2010)) the choice of $\kappa$ has to be made over the set of sustainable debt plans. But one should bear in mind that this latter set is not easily prespecified, and will depend on the future negotiations between the sovereign and foreign investors. This technical issue has been overlooked in the literature.

In view of such computational challenges, we attempt to get tighter debt bounds for sustainable debt paths numerically through an iterative procedure. Let $\mathcal{A}$ denote the space of total factor productivity $A$, $\mathcal{K}$ the space of physical capital $K$, and $\mathcal{B}$ the space of government budget-feasible debt $B$. Let $\mathcal{H}$ be the cartesian product $\mathcal{A} \times \mathcal{K} \times \mathcal{B}$. In what follows, we define an operator $\mathcal{D}$ in $\mathcal{H}$ whose fixed point is the set of equilibrium sustainable debt. To facilitate the definition of $\mathcal{D}$, we begin with the admissibility of $B$ with respect to a subset $Y \subset \mathcal{H}$.

For given value and policy functions $\tilde{W} = \left( W(\theta; \tilde{\Theta}(\theta)), \tilde{\Theta}(\theta) \right)$, where $\theta = (A, K, B)$, we say that vector $\psi = (K_+, l, \Delta, G, \tau, B_+)$ is admissible with respect to $Y$ if $\psi$ satisfies (P-3) for $\tilde{W}$ at all $\theta$ and $(A_+, K_+, B_+) \in Y$.

**Definition 4.4.** For a given set of equilibrium sustainable debt $Y \subset \mathcal{H}$, operator $\mathcal{D}$ is defined as

$$\mathcal{D}^W(Y)(A, K) = \left\{ B | \exists \psi \text{ that is admissible w.r.t. } Y \text{ at } (A, K) \text{ for } \tilde{W} \right\}.$$ 

In words, admissibility means that vector $\psi$ guarantees existence of optimal allocations for the household and the government satisfying feasibility, while the continuation value of implied debt choice $B_+$ falls within set $Y$.

Using arguments along the lines of Phelan and Stacchetti (2001), for every optimal value function $W(\theta; \Theta)$ and equilibrium policy $\Theta$ in (P-3), we can get the following.

1. $\mathcal{D}(\cdot)$ is monotone and preserves compactness.

2. If $Y \subseteq \mathcal{D}(Y)$, then $\mathcal{D}(Y) \subseteq B$.

3. $B$ is compact and the largest equilibrium set of sustainable debt $Y$ such that $Y = \mathcal{D}(Y)$. 

$\tilde{\Theta}(\theta)$ indicates the continuation value of the equilibrium policy.
4. If we define \( Y_{n+1} = D(Y_n) \) for all \( n \geq 0 \), and the equilibrium set of sustainable debt \( B \subset Y_0 \), then \( \lim_{n \to \infty} Y_n = B \).

With these preliminaries, we now pass to explain the numerical implementation of the algorithm. Further technical details can be found in the Appendix. As already discussed, the current sovereign needs to forecast the paths of taxation, debt issuance, and default decision by future governments, as well as the private sector’s optimal decision rules for these given policies. We therefore start the algorithm with the following objects: (i) an initial guess for the set of sustainable debt at every given \((A, K)\); (ii) an initial guess for the perceived taxation, debt issuance and default decisions as functions of the state of the economy; and (iii) an initial guess for the representative household’s optimal decision rules as equilibrium functions of the state of the economy.

We then solve the model under an inner and an outer loop. In the inner loop, for a fixed initial guess of perceived policies and the household’s policy and value functions, we compute problem (P-3) through value function iteration. This step yields a new set of policy and value functions for the government and the representative household, which are subsequently used to replace the perceived policy and value functions. We keep iterating until the policy and value functions solving problem (P-3) are sufficiently close to the previous inputed policy and value functions for the future government and the representative household.

In the outer loop, we take as given the converged policy and value functions from the inner loop, and update the set of budget-feasible debt policies for the sovereign by implementing operator \( D \). At advanced stages of the iteration process, the idea is to work with a relatively small borrowing set. We then go back to the inner loop and find the policy and value functions that solve problem (P-3). The process stops when we find convergence in both the inner and outer loops.

As an illustration, Figure 2 presents the computed equilibrium set of sustainable debt for a fixed value of the productivity level \( A \). The dots in Figure 2 represent grid points that are used to approximate the space of state variables. Note that the set of sustainable debt is increasing with the capital stock, but it does not have a well defined form. In the mid-area of the state space, we endogenously identify the cutoff value of default and allocate more grid points around that region to improve computational accuracy. The sovereign wants to default for every bond value above the cutoff level.

To conclude, the computation of an MPE must deal with some complex technical issues. While our algorithm draws on Abreu, Pearce, and Stacchetti (1990) and the numerical implementation to macroeconomic models found in Feng et al. (2014) and Feng (2015), all these
available methods fall short of what is required for the simulation of the present model of sovereign default. First, external debt, partial default, endogenous fiscal policy, production and investment make that the budget-feasible correspondence of sustainable debt $B$ can be hard to encapsulate within tight bounds; this correspondence may not have a well defined geometrical shape. We can narrow down numerically this set under good guesses for the perceived decision rules of the sovereign and the representative household. Second, default and time-consistent fiscal policies may generate kinks and non-concavities of the value functions. These functions are hard to approximate and there could be multiple maximizers. We estimate approximation errors numerically. And third, our characterization of the equilibrium law of motion involves the fixed point of two related value and policy functions; this iterative procedure is quite slow.

Figure 2: The correspondence of sustainable debt

5 The Baseline Economy

5.1 Calibration

For the calibration of parameter values, we adopt a two-step estimation procedure. In the first step, we estimate the shock process to match the persistency and volatility of output
in the data, and assign parameters to preference and production functions. The remaining parameters are estimated by the simulated method of moments (SMM).

**Functional forms** Preferences are represented by the following utility functions:

\[ u(c, G) = \frac{c^{1-\sigma}}{1-\sigma} + \gamma_g \frac{G^{1-\sigma}}{1-\sigma}, \]
\[ h(l) = \frac{\gamma_l l^{1+\chi}}{1+\chi}. \]

Observe that \( \sigma \) is the coefficient of relative risk aversion, and \( \chi \) defines the labor supply elasticity. Parameters \( \gamma_g \) and \( \gamma_l \) are the weights of public consumption and the disutility of labor in the one-period utility function.

Aggregate production is represented by the Cobb-Douglas function:

\[ F(K, L) = AK^\alpha L^{1-\alpha}. \]

The TFP shock follows an AR(1) process:

\[ \log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t} \]
\[ \varepsilon_{A,t} \sim N(0, \sigma_A^2). \]

The productivity loss upon the event of default is specified as \( \zeta(A) = \max (\zeta_1 A + \zeta_2 A^2, 0) \).

This captures the asymmetric output losses of Arellano (2008), and Chatterjee and Eyigun-gor (2012). A convex output loss lowers the incentive to default in high-output states, and improves the negative correlation of output with the sovereign bond spread.\(^6\)

**Parameter values** The following parameters \( \{\beta, \tilde{r}, \sigma, \chi, \gamma_g, \gamma_l, \alpha, \delta, \rho_A, \sigma_A, \kappa\} \) are commonly discussed in the literature, while their values are usually selected to match some basic data statistics. We set the discount factor \( \beta = 0.95 \) to match the capital-output ratio in the benchmark economy. We let \( \tilde{r} = 4\% \) for the annual interest rate of the foreign lending sector. We fix the degree of relative risk aversion \( \sigma \) to equal to unity, and let \( \chi = 0.32 \) so as to get a Frisch labor supply elasticity equal to 2.0. Public consumption carries a weight \( \gamma_g = 0.222 \) to attain an average government spending to output ratio close to 15 percent,\(^6\)

---

\(^6\)Based on extensive numerical experiments, our model is robust to different types of productivity loss function.
Table 3: Parameter values and targeted data statistics. Annual data.

<table>
<thead>
<tr>
<th>Calibrated parameters</th>
<th>Value</th>
<th>Targeted Data Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ): discount factor</td>
<td>0.95</td>
<td>Capital-output ratio</td>
</tr>
<tr>
<td>( \sigma ): risk aversion</td>
<td>1.0</td>
<td>Standard RBC value</td>
</tr>
<tr>
<td>( \chi ): labor elasticity</td>
<td>0.32</td>
<td>Frisch wage elasticity (2.0)</td>
</tr>
<tr>
<td>( \alpha ): capital share</td>
<td>0.34</td>
<td>Capital share in GDP</td>
</tr>
<tr>
<td>( \gamma_g ): public consumption</td>
<td>0.222</td>
<td>Government spending</td>
</tr>
<tr>
<td>( \gamma_n ): disutility of labor</td>
<td>5.905</td>
<td>Aggregate labor supply</td>
</tr>
<tr>
<td>( \rho ): persistence of TFP</td>
<td>0.95</td>
<td>GDP auto-correlation</td>
</tr>
<tr>
<td>( \sigma_A ): TFP shock</td>
<td>0.024</td>
<td>GDP std. deviation</td>
</tr>
<tr>
<td>( \kappa ): haircut</td>
<td>0.4</td>
<td>Cruces and Trebesch (2013)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters estimated by SMM</th>
<th>Value</th>
<th>Targeted Data Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu ): default penalty</td>
<td>0.3</td>
<td>Level of debt</td>
</tr>
<tr>
<td>( (\xi_1, \xi_2) ): productivity loss</td>
<td>((-0.71, 0.73))</td>
<td>Investment and output loss</td>
</tr>
<tr>
<td>( \pi_A ): persistence of productivity loss</td>
<td>0.2</td>
<td>Sovereign bond spread</td>
</tr>
</tbody>
</table>

which is the average level observed in our sample data. The labor weight \( \gamma_l = 5.905 \), which yields a fraction of labor supplied around 0.23.

We fix \( \alpha = 0.34 \) and \( \delta = 0.08 \). To match the auto-correlation and volatility of GDP, we let the persistence parameter \( \rho = 0.95 \), and the volatility parameter, \( \sigma_A = 0.016 \).

The predetermined haircut value \( \kappa \) is set to 0.40. Sturzenegger and Zettelmeyer (2008) find that the haircut ranges between 0.13 to 0.73, and Cruces and Trebesch (2013) come up with an average haircut value \( \kappa = 0.37 \) after debt restructuring.

For the remaining four parameters, \( \{ \nu, \pi_A, \xi_1, \xi_2 \} \), we use the method of simulated moments to match the following targeted data statistics: the level of sustainable debt in equilibrium, the drop in output and investment at about the time of default, and the sovereign bond spread; see Table 3.

#### 5.2 Equilibrium

Panel (a) of Figure 3 plots the debt cutoff value for default as a function of the capital stock \( K \). Observe that this cutoff value is increasing in \( K \) as well as in the productivity parameter \( A \). This suggests that the sovereign is less likely to default for high \( K \), and in boom periods. Indeed, the sovereign can most easily resort to taxation when TFP is high to smooth consumption. This policy tradeoff between taxation and borrowing is illustrated in panel (b) of Figure 3. The tax functions are increasing in the debt level \( B \), since there is a rising cost of financing public consumption through debt issuance. There is also the
government’s incentive to reduce the probability of default by switching from debt issuance to tax revenue.

Panel (c) of Figure 3 depicts $B_+$ as a function of $K$ and $B$. For given $K$, there is a sharp rise in $B_+$ as we reach the cutoff value of $B$ for default. The sharp increase in debt issuance is triggered by the productivity loss after partial default. Function $B_+$ presents another kink point once $B$ becomes sufficiently high. For a predetermined $\kappa$, the remaining debt after default may exceed the cutoff value of default in the immediate future period. This will force the sovereign to repeated default.

The decision to default also breaks the concavity of the value functions. In panel (d) of Figure 3, we plot the government’s value function for a fixed level of $K$. There are kinks for this payoff function as the level of debt increases. Hence, there is a sharp decrease in the bond price around the debt cutoff value for default. Default prompts a sudden drop in the expected payoff of the international creditor, which causes the bond price to decline steeply and the borrowing cost to skyrocket; see Figure 3(e)-(f).
5.3 Simulation

With the computed equilibrium policy and value functions for our baseline economy, we generate equilibrium sample paths under different initial conditions and productivity shocks.
Each simulation lasts for 50,000 periods, but we drop the first 40,000 periods to make sure that these paths reach the ergodic distribution, which appears to be unique.

### 5.3.1 Macroeconomic dynamics around default

As discussed in Section 2, we calculate the average evolution of our macroeconomic variables over a five-year window before and after default, and follow the same procedure for the corresponding model statistics. In Figure 4, the dashed lines refer to the data values and the solid lines refer to the simulated values. Recall that for our baseline economy, these key parameters \( \{\nu, \pi_A, \zeta_1, \zeta_2\} \) were estimated by the SMM with a view towards targeting some model predictions at the time of default.

![Figure 4: Macroeconomic dynamics of the baseline economy](image)

Our baseline model replicates well the macroeconomic dynamics summarized under the stylized facts in Section 2.

**Macroeconomic activity:** In our model, default happens when the economy has been hit by an unfavorable TFP shock. The model replicates the drop in investment and labor supply around sovereign default. On average, the share of total investment in GDP declines by 3.2% (from 17.3% to 14.1%) prior to the default, as opposed to 5.3% (from 23.11% to 17.89%) in the data. Similarly, hours worked per person decrease by 3.3% in the baseline
economy, as opposed to the 4.2% observed in the data. Hence, the model generates a smaller variability of the production factors at the cost of a greater variability of TFP. The share of total private consumption in GDP stays roughly constant as in the data. Our model cannot generate the decline in private consumption after default, which may be due to unequal fiscal policies in the model and the data (i.e., a fiscal friction).

**Fiscal policy:** In our model, the government runs a fiscal surplus to avoid the default cost. The average tax rate increases from 15% to 21% during default, and public spending stays flat. In our database, however, we can observe persistent fiscal deficits. Tax revenues and general government consumption are mostly flat before default. More specifically, the observed tax rate is about 15% and it fluctuates less than 3% within the five year interval prior to default, while public consumption increases slightly before default.

**Sovereign debt:** By considering partial debt repudiation, our model also replicates the hump shaped debt dynamics around the time of default. The debt-to-GDP ratio rises from 69% prior the default to 80% at the time of default, then gradually drops to 42.2% during the post default period. While in the data, the debt-to-GDP ratio increases from 57.5% to 78%, and then decreases to 53.1%.

**Sovereign bond spread:** In our simulations the borrowing cost rises steeply to about 4,000 basis points at the time of default. Then, it drops gradually to attain pre-default levels after five years from default. In our database, this average spread increases to about 4,200 basis points at the time of default. Then, its decline after default is not so pronounced as in the model.

We summarize all these predictions of the model in Table 4, where we report averages of these variables before and after default. As it stands out, the most salient problem is the rather low credit risk premium after default. In the next section we pursue a more detailed investigation of some of the determinants of the level and persistence of the risk premium. We find that changes in $\kappa$ and $\pi_A$ have a notable influence on the debt-to-GDP ratio and the sovereign bond spread.

Most of the literature assumes complete debt write-offs with shut-down of the foreign lending sector after sovereign default. Then, the debt-to-GDP ratio becomes zero after default. Obviously, these approaches necessarily generate low debt-to-GDP ratios along with exploding sovereign bond spreads. A recent literature considers long-term debt and
Table 4: Macroeconomic dynamics around default

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Overall</th>
<th>pre default</th>
<th>post default</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Baseline</td>
<td>Data</td>
</tr>
<tr>
<td>Average value</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP growth</td>
<td>1.98</td>
<td>-0.76</td>
<td>0.32</td>
</tr>
<tr>
<td>private consumption GDP</td>
<td>68.4</td>
<td>68.81</td>
<td>67.2</td>
</tr>
<tr>
<td>investment</td>
<td>21.03</td>
<td>17.1</td>
<td>23.11</td>
</tr>
<tr>
<td>hours worked per person</td>
<td>22.17</td>
<td>24.1</td>
<td>22.3</td>
</tr>
<tr>
<td>tax revenue</td>
<td>15.28</td>
<td>15.0</td>
<td>15.5</td>
</tr>
<tr>
<td>public consumption</td>
<td>14.06</td>
<td>13.6</td>
<td>14.6</td>
</tr>
<tr>
<td>debt to GDP ratio</td>
<td>57.22</td>
<td>57.9</td>
<td>57.5</td>
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<tr>
<td>sovereign spread (bpts)</td>
<td>913</td>
<td>1408</td>
<td>611</td>
</tr>
</tbody>
</table>

5.3.2 Cyclical co-movements

We also assess the performance of the model by looking at some simulated moments from the ergodic distribution. As shown in Table 5, the model approximates well the correlations of private consumption, investment and labor supply with GDP as compared to those observed in the data. The correlation of private consumption with GDP is 0.97 in the model as opposed to 0.93 in the data. The correlation of investment with GDP is 0.77 in the model as opposed to 0.85 in the data. The correlation of labor supply with GDP is 0.40 in the model as opposed to 0.39 in the data. The model also generates counter-cyclical tax policies and counter-cyclical bond spreads as observed in emerging economies.

As in the business cycle literature, the model generates a low volatility of consumption, even though it approximates well the volatility of investment relative to GDP. By borrowing from abroad, the government has an additional channel to smooth consumption. Consequently, we should expect a lower volatility in consumption relative to output and in the credit risk premium. Note that we are abstracting from the business cycle dynamics of the
Table 5: Business cycle statistics

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Baseline model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation with GDP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>private consumption</td>
<td>0.93</td>
<td>0.97</td>
</tr>
<tr>
<td>investment</td>
<td>0.85</td>
<td>0.77</td>
</tr>
<tr>
<td>labor</td>
<td>0.39</td>
<td>0.40</td>
</tr>
<tr>
<td>tax</td>
<td>-0.33</td>
<td>-0.11</td>
</tr>
<tr>
<td>bond spread</td>
<td>-0.62</td>
<td>-0.36</td>
</tr>
<tr>
<td>Private consumption volatility / GDP volatility</td>
<td>1.23</td>
<td>0.66</td>
</tr>
<tr>
<td>Investment volatility / GDP volatility</td>
<td>2.66</td>
<td>2.02</td>
</tr>
<tr>
<td>Average debt/GDP ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default probability</td>
<td>2.78%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Average bond spread</td>
<td>1.86%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Bond spread volatility</td>
<td>0.78%</td>
<td>0.39%</td>
</tr>
</tbody>
</table>

world economy, and simply assume an exogenous interest rate for the foreign lending sector. The model produces an average debt-to-GDP ratio of 67.4% as compared to 70% in our data sample. As already stressed, this is significantly larger than the mean debt ratios typically obtained in quantitative models of sovereign default with full debt repudiation. The frequency of default, however, is much lower in our model than in the data. Again, we should note that our modeling of the foreign sector is overly simplified; sovereign default is often triggered by rare events (e.g., global financial crises), or sudden changes in external interest rates.

6 Comparative Statics

We now run some counterfactuals after perturbation of some key parameter values which may vary across countries. This type of sensitivity analysis appears to be missing in the literature, and we lack a broad understanding of the impact of some parameters on economic aggregates at the time of default. For simplicity, we focus on changes in the debt-to-GDP ratio and the sovereign bond spread.

\footnote{For example, Arellano (2008) obtains a mean debt ratio of 6% of GDP in a model with exogenous output shock and default cost, while Mendoza and Yue (2012) generate an average debt ratio of 23% in a model considering production and endogenous default cost.}
6.1 The size of the haircut $\kappa$

As in the standard quantitative sovereign default literature, we now increase the haircut to 100%. Then, the optimal debt-to-GDP ratio upon default goes from 80% to 48%, and the sovereign bond spread goes from 4,000 to about 40,000 basis points; see Figure 5(b). Complete default is very attractive in that the government can off-load its debt obligations. Such full debt dispossession is taken into account by foreign creditors, who demand a much larger credit risk premium—increasing further the incentive to sovereign default. Tax revenues go down after the “fresh start”. Therefore, the government is motivated to default even at low debt levels, and foreign creditors anticipate such capital losses. Rather than trying to avoid default through fiscal policy, the sovereign lowers taxes and issues new debt after default.

These reinforcing effects are also patent as we move to the other extreme. Figure 5(c) portrays the equilibrium dynamics for $\kappa = 0.20$. A smaller haircut props up the debt-to-GDP ratio and decreases the sovereign bond spread. External financing is now cheaper to smooth public consumption. Sovereign default becomes less attractive, and a tight fiscal policy would avoid default.

In summary, the debt-to-GDP ratio and the sovereign bond spread are fairly sensitive to parameter $\kappa$. Limiting the haircut to $\kappa = 1$ in our model would offer a rather distorted view of the effects of sovereign default on our macroeconomic aggregates. In practice, the range of variation of the haircut is very wide [e.g., Edwards (2015), Sturzenegger and Zettelmeyer (2008)]. As a policy prescription, our model then calls attention to the unintended consequences of sizable haircuts by triggering serial default. As predicated by the World Bank, many poor countries have been granted rather generous programs of debt relief, but one should not ignore the negative incentive effect of these policies to encourage future default rather than sound fiscal policies.
Figure 5: The size of the haircut $\kappa$

(a) $\kappa = 0.4$  
(b) $\kappa = 1.0$  
(c) $\kappa = 0.2$
6.2 The TFP loss after default $\zeta$

Figure 6: The TFP loss after default $\zeta$

Figure 6 presents simulated data for changes in the loss of TFP after default. A bigger TFP loss makes sovereign default more costly, and enhances the role of fiscal policy to avoid default. As the TFP loss $\zeta$ is increased to the range between 3% and 20%, the primary fiscal surplus jumps to 15% of GDP. At the time of default, the debt-to-GDP ratio shoots to 130% from about 80% in the baseline economy, and the sovereign bond spread goes down by over 1,000 basis points.

Conversely, a smaller TFP loss reduces the government’s incentive to impose tight fiscal measures. While the economy is in a better solvency position for a lower TFP loss $\zeta$ after default, the debt-to-GDP ratio goes down and there is a rather small change in the sovereign bond spread. Indeed, a low cost of default together with a sizable haircut may result in serial default. Therefore, the sovereign bond spread could be quite insensitive to changes in
the TFP loss after default because of the added incentive to default under better economic conditions.

6.3 The persistence of the TFP loss after default $\pi_A$

This is another parameter which is hard to calibrate because the persistence of the TFP loss will depend upon the nature of the economic situation. Hence, the probability of recovery $\pi_A$ can vary across countries and time periods. We start in Figure 7(b) with a more persistent TFP loss, $\pi_A = 0.1$. Then, this increase in the cost of default prompts a much tighter fiscal policy and a higher sustainable debt-to-GDP ratio as well as an increased sovereign bond spread. As we can observe, the credit risk premium is quite sensitive to changes in $\pi_A$. A lower $\pi_A$ increases the cost of default as well as the debt-to-GDP ratio, since it may take a long time for the economy to recover.

On the other hand, Figure 7(c) portrays changes in our macroeconomic variables under a higher value of $\pi_A$. This could be thought as a prototypical case of “gambling for redemption” [cf. Conesa and Kehoe (2017)] in which fiscal policy remains quite insensitive to the event of default. This lack of response to default extends to other macroeconomic variables. The credit spread, however, may not be decreased because default becomes much cheaper and would occur more frequently.
Figure 7: The persistence of the TFP loss after default $\pi_A$

(a) $\pi_A = 0.2$

(b) $\pi_A = 0.1$

(c) $\pi_A = 0.3$

6.4 The intangible utility loss $\nu$

Lastly, we study the impact of the non-pecuniary penalty $\nu$ on the macroeconomic dynamics and the sovereign default decision. This one-time loss helps to calibrate the cost of default. Hence, we expect that a higher $\nu$ will increase the debt-to-GDP ratio, and may have a much smaller effect on the credit risk premium. Of course, a higher $\nu$ enhances the role of fiscal policy to avoid default. In fact, for very high values of $\nu$ [Figure 8(b)] the fiscal policy adjustment may start a few years before default, whereas for very low values of $\nu$ the tax revenue and general consumption expenditure will remain flat around default. As in previous cases, contractionary fiscal policies can discourage investment and labor supply.
7 Extensions

Our baseline economy has offered some clues on the effects of sovereign default on the macroeconomy for given changes in parameter values. We now incorporate some extensions to this tractable framework. We assume that TFP declines before default, and acknowledge the existence of fiscal and financial frictions. All these extensions can substantially change our quantitative results. TFP is a main driver of economic activity, while fiscal deficits may accelerate default. Financial frictions will increase the cost of default through a higher sovereign bond spread.

Most models of sovereign default assume that the output loss happens right after default. Paradoxically, in some extended variants of these models, economic activity could go up before default because of intertemporal substitution effects. As discussed in Section 2, however, TFP may start declining 5 years before default. Then, it goes down to about 91
percent of the peak value, and begins a slow recovery after default. This gradual decline in TFP may actually be blamed for the implosion in consumption, investment, and hours worked in these default episodes.

While a fiscal primary surplus may be an optimal response to avoid default, in reality countries follow expansionary fiscal policies at the time of default; usually, the share of the tax revenue in GDP stays mostly flat. Hence, there may be some fiscal policy inertia, which may radically change the predictions of the model on economic activity. Taxation could significantly distort the paths of consumption, investment, and employment.

After default, credit default swaps for external debt go up well beyond domestic rates; see Figure 1. This added risk premium on external debt appears to be long lasting, and could be a reflection of the “investment home bias” after a reputation loss upon default. We let the intangible loss of utility parameter, \( \nu = 0 \), and incorporate this lingering effect on the financial market to add to the sovereign bond spread.

**The time profile of the TFP loss:** Empirically, we have observed that TFP starts to decline for debt levels close to 85\% of the default cutoff value. This threshold value for the declining of TFP is roughly reached about five years before default. There are various economic reasons for this gradual decline in TFP. A high probability of default may lead to higher interest rates, and may discourage bank lending, the arrival of new firms, investments in physical and organizational capital, and so forth; cf., Arellano, Bai, and Bocola (2017), Bocola (2016), and Perez (2019). We will therefore fit the evolution of TFP to the data; see Figure 9(a). Alternatively, we could have assumed that a high probability of sovereign default may foreshadow a loss of physical capital to the economy. However, this loss of physical capital around default will foster savings and investment as confirmed in our numerical simulations. These investment patterns are at odds with the data.

**Fiscal policy inertia:** To circumvent sharp changes in taxation and fiscal balances as prescribed by optimal fiscal policies, we can impose an upper bound on the tax rate, \( \tau \), or limit its rate of growth. In fact, we consider a much simpler exercise in which the tax rate is set to \( \tau = 0.16 \), and total government expenditure is also constant and equals 15\% of GDP. Therefore, this constant policy generates a one-percent primary fiscal surplus at all times. As already stressed, this policy should have more neutral effects on the dynamics of consumption, investment, and employment.
Financial friction: In our financial friction the government must pay a premium for borrowing after default. The price of the bond in the post-default era includes a wedge $\eta$ which captures the increased spread for external debt:

$$q^x = (1 - \eta) \frac{\mathbb{E}(1 - \Delta_+ \cdot \kappa)}{1 + \bar{r}}.$$ 

As with the TFP loss, the additional cost of borrowing $\eta$ will be lifted with probability $\pi_A$ at every date after default. This prolonged cost of default from borrowing in international markets can be documented from several sources; e.g., see Figure 1. From the Emerging Market Bond Index Global (EMBIG), credit spreads for past defaulters are higher than for non-defaulters by 2-2.4 percentage points [cf. Asonuma (2016)]. Hence, this financial friction supports a more persistent credit spread after default.

Wage rigidity: The loss in both TFP and physical capital investment will affect the productivity of labor. But in reality, the real wage hardly goes down in these default episodes; e.g., see Na et al. (2018). Figure 9(b) plots the evolution of labor productivity against the observed real wage. For the computation of labor productivity in our Cobb-Douglas production, we plug in the laws of motion of TFP, physical capital, and labor from our dataset. We can see that at the time of default the real wage is about 25 percent higher than labor productivity. About half of this wage gap is due to the loss in labor productivity because of the declines in TFP and physical capital, and the other half is due to the increase in the real wage over time. We incorporate this wage-productivity gap into this extended version of our model.

Figure 9: The gradual decline in TFP and the wage-productivity gap
Figure 10 displays the evolution of our economic variables in both the baseline economy of Section 5 and the economy embedding the above four extensions. As we can see, there are not many noticeable changes in the behavior of these economic aggregates. In fact, we get a better model’s performance regarding the sovereign bond spread and the variations in investment and employment. Hence, this numerical exercise confirms that the main predictions of the model do not hinge upon the intangible utility loss $\nu$ and the distortionary effects of the optimal fiscal policy. In this new model, the decline in consumption, investment, and employment should be attributed to the gradual loss in TFP and the downward rigidity of the real wage. The added cost of default comes from the exogenous increase in the credit risk premium.

Figure 10: The baseline economy and an extended economy with a gradual decline in TFP, fiscal and financial frictions, and a wage-productivity gap

8 Concluding Remarks

In this paper we present a computational framework for sovereign default. We introduce a production sector with physical capital and labor, income taxes and public consumption, and partial debt repudiation. We model the cost of default as a persistent loss in TFP that may predate the event of default. These extensions of the prototypical model of sovereign default are in line with standard practices in macroeconomic modeling and data gathering as reflected in the National Accounts.

We find that under various conditions our model can generate realistic values for the debt-to-GDP ratio and the sovereign bond spread, and for the evolution of some real economic aggregates (e.g., consumption, investment, and employment). For our baseline economy,
the observed debt-to-GDP ratio entails a rather high and persistent default cost. Hence, the productivity loss has to be substantial, sufficiently persistent, and it may need to occur before the event of default. Of course, we are circumventing the frictions and transmission mechanisms underlying these economic and financial crises which are dependent on economic conditions across countries. Our approach, however, should be viewed as a useful approximation to a rather complex problem, and provides a simple yardstick to evaluate some micro-founded models of sovereign default. Most models of debt repudiation assume that the output loss will happen after default. This could be at odds with the observed evolution of consumption, investment, and employment before the occurrence of default. We have also ruled out the modeling of default as a negative shock to the physical capital stock.

As observed in several other economic episodes, wage rigidities appear to be behind the lowering of employment at the time of default. Fiscal budget deficits in our framework occur under complete default, and for rather small and less persistent TFP losses. The model then calls attention to a fiscal friction which has been largely unexplored in the literature of sovereign default. We also attest the existence of a financial friction: external financing becomes relatively more costly after default. We summarize this friction as an added wedge in the pricing of the foreign bond.

In our framework, the debt-to-GDP ratio and the credit spread are fairly sensitive to the haircut and to the persistence of the productivity loss. Under complete default, the debt-to-GDP ratio plummets while the credit spread may skyrocket. Here, investors face a full debt wipe-out, and the sovereign is always tempted to default. Also, under high persistence of the cost of default, the debt-to-GDP ratio would be high upon default. Hence, a high debt together with a low probability of economic recovery generates a long-lasting credit spread. On the other hand, the credit spread is less sensitive to the magnitude of the TFP loss upon default; under a small TFP loss the government follows a more accommodating fiscal policy and becomes more inclined to default.

These results call into question full debt relief programs and HIPC initiatives by the IMF and World Bank. Such recommendations encourage serial default at the expense of prudent fiscal policies. In our framework, the haircut emerges as an important policy tool towards fiscal discipline and to monitor the frequency of defaults. Policy makers should certainly evaluate the unintended economic consequences of debt relief.

\[8\] Krueger, Mitman, and Perri (2016) also assume a drop in TFP as a result of the recent financial crisis after the bursting of the housing bubble. Cerra and Saxena (2008) document that some economic and political crises may trigger drops in the trend rather than a deviation from trend. These two papers only consider TFP losses after the onset of the crisis.
The large variation of debt-to-GDP ratios and sovereign bond spreads across countries appears quite puzzling. Again, more often than not, researchers and practitioners underestimate the cost of default.\textsuperscript{9} Our various numerical exercises should further our understanding of the magnitude of these costs across countries. While a large number of less developed economies have been exposed to repeated sovereign defaults, advanced economies are supported by strong institutions and sophisticated financial systems, and so they may have to face very adverse costs upon default. As these advanced economies are now dealing with mounting sovereign debts, the probability of default could nevertheless become significant in a medium term.

\textsuperscript{9}President Trump notoriously stated in 2016: \textit{I would borrow, knowing that if the economy crashed, you could make a deal. And if the economy was good, it was good. So, therefore, you can’t lose.} These beliefs not only ignore the impact of a U.S. default on the credit spread, but also the possible costs of default on economic productivity as manifested in the Lehman Brothers collapse. U.S. bonds are still priced as risk-free assets in spite of the high debt levels.
References


Online Appendix

Data Sources

Table 6 describes data sources of our data set for cross-country event studies. Table 7 summarizes the list of countries, default episodes, and available variables in the analysis.

Table 6: Data Sources

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definition</th>
<th>Frequency</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TFP$</td>
<td>TFP</td>
<td>annual</td>
<td>World Bank</td>
</tr>
<tr>
<td>$Y$</td>
<td>Real GDP</td>
<td>annual</td>
<td>World Bank</td>
</tr>
<tr>
<td>$C$</td>
<td>Private consumption</td>
<td>annual</td>
<td>World Bank</td>
</tr>
<tr>
<td>$G$</td>
<td>General Government consumption</td>
<td>annual</td>
<td>World Bank</td>
</tr>
<tr>
<td>$I$</td>
<td>Investment</td>
<td>annual</td>
<td>World Bank</td>
</tr>
<tr>
<td>$IM/EX$</td>
<td>Import / Export</td>
<td>annual</td>
<td>World Bank</td>
</tr>
<tr>
<td>$L$</td>
<td>Employment</td>
<td>annual</td>
<td>World Bank</td>
</tr>
<tr>
<td>$B_d$</td>
<td>External debt / GDP</td>
<td>annual</td>
<td>Panizza (2008), supp. w/</td>
</tr>
<tr>
<td>$B_x$</td>
<td>Domestic debt / GDP</td>
<td>annual</td>
<td>Rinhart and Rogoff (2010).</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tax revenue</td>
<td>annual</td>
<td>World Bank</td>
</tr>
<tr>
<td>$FBY$</td>
<td>Fiscal balance</td>
<td>annual</td>
<td>World Bank</td>
</tr>
<tr>
<td>$r_d$</td>
<td>Real interest rate</td>
<td>annual</td>
<td>World Bank</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Sovereign debt rate</td>
<td>annual</td>
<td>J. P. Morgan EMBI+</td>
</tr>
</tbody>
</table>

The default episodes are based on Yeyati and Panizza (2011). The debt structure data is provided by Panizza (2008), whose study covers developing countries.\footnote{In Panizza (2008), the external debt is defined as the debt owned to non-residents. This is consistent with the definition adopted by The External Debt Statistics: Guide for Compilers and Users, published by the BIS, Eurostat, IMP, OECD and the World Bank.} We supplement this data with Reinhart and Rogoff (2011). The sovereign bond spread is taken from the J. P. Morgan Emerging Markets Bond Index (EMBI+) for Argentina (1994Q1-2002Q2) and Ecuador (1996Q2-1999Q2). For all the other countries in our sample, we obtain the aggregate sovereign bond spread from interest data of the Database of Fiscal Space complied by Kose et al. (2017).\footnote{The data is available at http://www.worldbank.org/en/research/brief/fiscal-space.} We get TFP, GDP, consumption, current account balance, fiscal balance, investment, government spending, taxation and the real interest rate data from the World Bank Open Data.\footnote{see https://data.worldbank.org/}
Table 7: List of countries and variables included in the event analysis

<table>
<thead>
<tr>
<th>Sovereign default</th>
<th>Available series</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina 2002Q2</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, \tau, FBY, r_d, r_x$</td>
</tr>
<tr>
<td>Ecuador 1999Q3</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, FBY, r_d, r_x$</td>
</tr>
<tr>
<td>Indonesia 1998Q3</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, \tau, FBY, r_d$</td>
</tr>
<tr>
<td>Pakistan 1998Q3</td>
<td>$Y, C, G, I, IM/NX, L, B_d, B_x, FBY, \tau$</td>
</tr>
<tr>
<td>Russia 1998Q4</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, \tau, FBY, r_d$</td>
</tr>
<tr>
<td>South Africa 1993Q1</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, \tau, r_d$</td>
</tr>
<tr>
<td>Thailand 1998Q1</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, FBY, r_d$</td>
</tr>
<tr>
<td>Ukraine 1998Q4</td>
<td>$Y, C, G, I, IM/NX, TFP, L, FBY, r_d$</td>
</tr>
<tr>
<td>Uruguay 2003Q2</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, \tau, FBY, r_d$</td>
</tr>
<tr>
<td>Venezuela 1995Q3</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, FBY, r_d$</td>
</tr>
<tr>
<td>Venezuela 1998Q3</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, FBY, r_d$</td>
</tr>
<tr>
<td>Mexico 1998Q2</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, FBY, r_d$</td>
</tr>
<tr>
<td>Moldova 2002Q2</td>
<td>$Y, C, G, I, IM/NX, TFP, L, B_d, B_x, \tau, FBY, r_d$</td>
</tr>
<tr>
<td>Greece 2012</td>
<td>$Y, I, TFP, L$</td>
</tr>
<tr>
<td>Iceland 2008Q3</td>
<td>$Y, I, IM/NX, TFP, L, r_d$</td>
</tr>
</tbody>
</table>
Proofs

Proposition 4.2:

Proof. The existence of $B$ follows the fact that $B_t = 0$, for $t = 0, 1, \ldots$, is a feasible choice for the government.

As is well known [cf. Hernandez and Santos (1996)], the existence of an optimal solution for the government implies that the sum of the discounted stream of tax revenues must be well defined: $E\left(\sum_{t=0}^{\infty} \frac{T_t}{(1+\tilde{r})^t}\right) < \infty$. Hence, every sustainable debt path must obey the transversality condition: $\lim_{t \to \infty} E\left(\sum_{t+1} B_{t+1} \frac{B_{t+1}}{(1+\tilde{r})^{t+1}}\right) = 0$. This condition ensures the boundness of the set $B$.

The set $B$ would be close if the utility and production functions are continuous in the product topology of $\mathbb{R}^\infty$. For more general cases, we must rule out private consumptions or public consumptions approaching zero. This may happen when the inherited debt obligation $B_t$ is sufficiently large. In such case, the set $B$ could be open as the economy can be arbitrarily close to but cannot reach a pre-specified debt limit. For if not, under the Inada condition the social welfare function is not well-defined. With the option to default under a finite default cost, it is always a better alternative for the government to default once it reaches that debt level. Since our economic environment guarantees that the cost of default is bounded, this immediately rules out the possibility that the set $B$ is open.

We have therefore established the existence of a compact set $B$.

Proposition 4.3:

Proof. For given government’s policies $\tilde{\Theta}$, and transversality condition, the solution to household’s optimization problem (P-1) can be characterized by (4, 5). The prices are competitive and characterized by equations (12, 13 and 20).

In a Markov perfect equilibrium, the perceived government policies must be identical to actual policies: $\tilde{\Theta} = \Theta$. And these policies are invariant functions of the state variables.

After plugging in these conditions into the government’s problem (P-2), we get the above formulation (P-3).
Numerical Algorithm

Our algorithm proposes a new method to characterize the endogenous feasible set of sustainable debt. The key idea is to implement operator $\mathbb{D}$ by generating a sequence of sets, whose limit converges to the equilibrium set of sustainable debt; see step 3 below for details. We refine the method of progressive grid search optimization and pre-loaded interpolation to improve the efficiency of computation. We also apply several recently developed computational techniques, including endogenous adaptive grids, and triangular interpolation [cf. Brumm and Grill (2014)]. These methods are easy to carry out and are quite relevant for related applications. To speed up the computation, we also use parallel computing in which each computing unit deals with one particular value of $(A, K, B)$. Our algorithm is implemented in C++ and MPI in a high performance cluster (HPC) with 7232 Intel Xeon cores in 452 nodes with 64GB RAM per node.

We compute the government’s problem using value function iteration.\footnote{Bachmann and Bai (2013) use value function iteration to compute the MPE in a setting with endogenous government spending and taxation, and no government borrowing.} We include the current default choice $\Delta$ in the vector of state variables to ease the computation burden of the non-convexity introduced by the decision to default.

Using Tauchen (1986) method, we discretize the AR(1) process for the TFP shock by equally spaced grid points, $\Omega_A = \{A_1, ..., A_{N_a}\}$. For capital $K$, we use equally spaced points $\Omega_k = \{k_1, ..., k_{N_k}\}$ to approximate $K = [k_\varepsilon, 1.5 \cdot k_{ss}]$, where $k_\varepsilon$ is a small positive number to circumvent the Inada condition, and $k_{ss}$ is steady state value of capital in a deterministic counterpart of the benchmark model with zero government tax, and no debt. We find the endogenous space of government debt $B(A, K)$ and generate endogenous adaptive grids $\Omega_b$ that are several times much more dense for the region around the cutoff value of sovereign default.

**Step 1:** We start the algorithm with the following guess of equilibrium functions including: pricing function: $\psi_q^{(0)}(\theta)$; policy functions: $\{\phi_c^{(0)}, \phi_k^{(0)}, \phi_b^{(0)}, \phi_l^{(0)}, \phi_r^{(0)}, \phi_G^{(0)}, \phi_\Delta^{(0)}\}(\theta)$; and value function: $W^{(0)}(\theta)$. These are defined over the initial grid points $\Omega_A \times \Omega_k \times \Omega_b^{(0)}$. Note that we use $\Omega_b^{(0)}$ to approximate the initial guess for the equilibrium set of sustainable debt: $B^{(0)}$. The initial guess of equilibrium functions is obtained using a homotopy. More specifically, we obtain the above guess by solving a sequence of simplified and easier to compute models, converging to our benchmark. For example, we start with a model without any financial or political friction, full capital depreciation, fixed government spending and taxation. We solve
this model by backward induction and use this “educated guess” as the initial condition to start our computation algorithm.

**Step 2:** We solve for the government’s problem, while taking into account the impact of its policies on household’s optimal choices.\(^\text{14}\)

\[
W^{(1)}(\theta) = \max_{\{K, l; G, \tau, B_+\}} \left\{ u(c, G) - h(l) - \Delta \nu + \beta \mathbb{E}(W^{(0)}(\theta_+)) \right\} \tag{28}
\]

subject to

\[
c + i + G + (1 - \Delta \kappa) \cdot B \leq AF(K, l) + \frac{\mathbb{E} \left( 1 - \phi_{\Delta_+}^{(0)} \cdot \kappa \right)}{1 + \bar{r}} \cdot B_+ \tag{29}
\]

\[
(1 - \delta) \cdot k + i \geq k_+ \tag{30}
\]

\[
G + (1 - \Delta \kappa) \cdot B \leq \frac{\mathbb{E} \left( 1 - \phi_{\Delta_+}^{(0)} \cdot \kappa \right)}{1 + \bar{r}} \cdot B_+ + \tau \cdot (AF(K, l) - \delta K) \tag{31}
\]

\[
u_c = \beta \cdot \mathbb{E} \left[ 1 + (1 - \phi_{\tau_+}^{(0)}) \cdot (F_{k+} - \delta) \right] \cdot u_c \left( \phi_{\tau_+}^{(0)}, \phi_{G_+}^{(0)} \right) \tag{32}
\]

\[
(1 - \tau) \cdot w \cdot u_c = h_l \tag{33}
\]

\[
B_+ \in B^{(0)}(A, K). \tag{34}
\]

There are kinks in the policy and value functions generated by the default decision and the time inconsistency, which complicate the computation of the optimization problem above. We concoct a progressive grid search optimization method to reduce the computation cost and to improve the accuracy. In particular, we first define a modestly dense set of grid points \(\Psi^{(0)}_k \times \Psi^{(0)}_b\) for \(K \times B\). We solve for the optimization problem defined above by imposing \((K_+, B_+) \in \Psi^{(0)}_k \times \Psi^{(0)}_b\). Once we find the optimizers \(K^{*(1)}_+, B^{*(1)}_+\), we then create new grid points around its neighborhood \(\Psi^{(1)}_k \times \Psi^{(1)}_b = [K_k^{*(1)} - \varepsilon_k, K_k^{*(1)} + \varepsilon_k] \times [B_k^{*(1)} - \varepsilon_b, B_k^{*(1)} + \varepsilon_b]\). Here \(\varepsilon_k\) and \(\varepsilon_b\) are small positive numbers. We re-do the optimization by imposing \((K_+, B_+) \in \Psi^{(1)}_k \times \Psi^{(1)}_b\), which yields improved optimizers \(K^{*(2)}_+, B^{*(2)}_+\). We create new grid points \(\Psi^{(2)}_k \times \Psi^{(2)}_b\) and repeat this process several times.

For the choice of \((K_+, B_+) \in \Psi_k \times \Psi_b\), it is very likely that \((K_+, B_+) \notin \Omega_k \times \Omega_b\). We

---

\(^{14}\)Given the utility function and production function specified in Section 5, we have \((1 - \tau) = \frac{\gamma^c - \gamma^l}{\gamma(1 - \alpha)\kappa} l^{\alpha + \chi}\) from (26). From the aggregate resource constraint (22) and government budget constraint (24), we have \((1 - \tau)(y - \delta K) = c + K_+ - (1 - \delta)K\). We can solve for \(l\) by combining these two equations, which yields

\[
l = \left( \frac{c + K_+ - (1 - \delta)K}{\gamma l^{\alpha}} \cdot c^{-\sigma(1 - \alpha)} \right)^{1/\alpha}, \quad \tau = 1 - \frac{\gamma^c - \gamma^l}{\gamma(1 - \alpha)\kappa} l^{\alpha + \chi}, \quad \text{and} \quad G = \tau(y - \delta K) + q \cdot B_+ - (1 - \Delta) \cdot B.
\]
use triangular interpolation to obtain the value of $W^{(0)}(\theta_+)$. This interpolation improves computational performance, particularly for the region where government switches to default.

The interpolation requires to identify the location of $(K_+, B_+) \in \Psi_k \times \Psi_b$ on the grid of $\Omega_k \times \Omega_b$. This is a rather costly computation by itself. We must repeat this process for any given $\theta = (A, K, B, \Delta)$, and all possible choices for $(K_+, B_+) \in \Psi_k \times \Psi_b$. To speed up computations and get higher accuracy, we apply the method of pre-loaded interpolation. We find the location of all possible $(K_+, B_+) \in \Psi_k \times \Psi_b$ before we start any optimization. We use a matrix $\Upsilon_{k,b}$ to store their coordinates on the set of grid points. For each interpolation, we retrieve the information from this giant matrix, which saves time for locating $(K_+, B_+)$ on the space of $\Omega_k \times \Omega_b$ repeatedly.

**Step 3:** The solution to the optimization problem in the previous step yields updated pricing function $\psi_q^{(1)}(\theta)$, policy functions $\left\{\phi_c^{(1)}, \phi_{k_+}^{(1)}, \phi_{b_+}^{(1)}, \phi_{l}^{(1)}, \phi_{r}^{(1)}, \phi_{G}^{(1)}, \phi_{\Delta}^{(1)}\right\}(\theta)$, and value function $W^{(1)}(\theta)$. Given the initial set $B^{(0)}(A, K)$, the set of $B^{(1)}(A, K)$ is obtained by keeping all points $B \in B^{(0)}(A, K)$ such that there exists $\{K_+, B_+\} \in K \times B^{(0)}(K_+, B_+)$, together with corresponding price function $\psi_q^{(1)}$, policy function $\phi^{(1)}$ and value function $W^{(1)}$, that solve the problem (P-4) for given values of $\{A, K\}$.

From the updated value function $W^{(1)}(\theta)$, we identify the region where the government is willing to default. Then we redefine the set $K \times B^{(1)}$ with $\Omega_k \times \Omega_b^{(1)}$, which are endogenous adaptive grids that allocate more basis points around the region of the default cutoff.

**Step 4** We iterate until convergence of all functions and sets is obtained: $\|W^{(1)} - W^{(0)}\| < \varepsilon^v_{tol}$, $\|\phi^{(1)} - \phi^{(0)}\| < \varepsilon^\phi_{tol}$, $\|\psi_q^{(1)} - \psi_q^{(0)}\| < \varepsilon^\psi_{tol}$, and $d_H(B^{(1)}, B^{(0)}) < \varepsilon^B_{tol}$, where $\varepsilon^v_{tol}$, $\varepsilon^\phi_{tol}$, $\varepsilon^\psi_{tol}$, and $\varepsilon^B_{tol}$ are some pre-specified small positive numbers, and the distance over these sets $d_H(\cdot, \cdot)$ is dictated by the Hausdorff metric.